Tetraquarks: Mesons with two b quarks

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Abstract

We look for the existence of stable (by strong interactions) tetraquarks, mesons containing two heavy antiquarks and two light quarks, specifically a $|\bar{b}\bar{b}qq\rangle$ state. This state is viewed as two $|\bar{b}q\rangle$ particles bound by a potential due to the exchange of the four light pseudoscalar and vector mesons, namely the π, η, ρ and ω. Couplings of these particles are obtained by the use of heavy quark symmetry. We find that the $I = 0$, $J = 1$ state is bound for a large range of parameters, while all other states are not bound (the $I = 1$, $J = 0$ system may be bound for some parameters at the extreme range of their allowed values). We discuss the reasons why the techniques used cannot be extended to mesons containing two c quarks or a c and b quark.

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I. INTRODUCTION

In addition to the usual three quark baryons and quark-antiquark mesons, hadrons containing one or more heavy quarks may occur in other configurations, stable against decay by strong interactions. Baryons with one heavy antiquark and four light quarks, *i.e.* pentaquarks [1], as well as baryons with two heavy antiquarks and five light quarks, *i.e.* heptaquarks [2], have been considered. Mesons containing two heavy antiquarks and two light quarks, tetraquarks [3–6], have also been studied. The approach used in the last two references was to consider a tetraquark as two mesons, each containing a heavy antiquark, bound to each other through the exchange a π meson.

In this work we shall look for bound states of mesons containing two $\bar{b}$ quarks and two light u or d quarks, namely a $(\bar{b}\bar{b}qq)$ system. As mentioned above we shall view such a possible system as a bound state of two B mesons, two B^* mesons, or a BB^* combination. It turns out that the bound states are linear combinations of these configurations. The binding potential will be due to the exchange, in addition to the π, of the η, ρ and ω mesons. The relevant coupling constants are obtained in Section II using heavy quark symmetry [7]. The Schrödinger equations and their solutions are discussed in Section IV. We find that bound states may exists for the light quarks in a symmetric color combination (presented in Section III); this is the opposite of the conclusions from bag model arguments [3,6]. For a wide range of parameters we find that a bound state exists for the system with total isospin $I = 0$ and total angular momentum $J = 1$. The range of binding energies below a possible decay to B and B^* mesons is about 650 MeV to 250 MeV. The $I = 1$ and $J = 0$ is unlikely to be bound.

Binding of states containing two c quarks or a c and a b quark are discussed in Section V. Our methods cannot be extended to the study of these states. As $M_{D^*} - M_D \sim m_\pi$ and for some charge combinations $M_{D^*} - M_D > m_\pi$ we cannot use the multi-channel approach in which we neglect the mass difference of the heavy pseudoscalar and vector mesons. Diagrams where the π is exchanged between a D and a D^* result either in a weak very long range

potential or in an imaginary one, indicating that one should treat the DD^* state as a three body $DD\pi$ system.

II. INTERACTION POTENTIAL

We shall be interested in obtaining the potential between two heavy mesons. In the heavy quark limit [7] the pseudoscalar and vector mesons are degenerate in mass and the bound state problem has to be treated as a multichannel one. As in this limit the masses of the incident and final pairs are equal and the energy of each of these states is just the sum of the two heavy masses, we are also allowed to use non-relativistic methods. These approximations are adequate for mesons containing $\bar{b}$ quarks but, as we will discuss, are likely to break down for systems with lighter quarks.

A. Heavy-light coupling constants

As in Ref. [5] we shall use the formalism of heavy quark symmetry and of the nonlinear chiral model [8] to obtain the coupling constants for the exchange of light mesons. With B^a and B_μ^{*a} denoting the fields for the pseudoscalar B and vector B^*, a being an isotopic spin index, we can define a heavy, fixed four–velocity V^μ field H^a,

$$H^a = \left(\frac{1 + V \cdot \gamma}{2}\right)(i\gamma^5 B^a + \gamma \cdot B_\mu^{*a}), \quad \bar{H} = \gamma^0 H^\dagger \gamma^0 \,. \tag{1}$$

The free Lagrangian for H is

$$\mathcal{L}_{free} = -iM\, V^\mu \mathrm{Tr}\left(H \partial_\mu \bar{H}\right) \,. \tag{2}$$

The light Goldstone pions and the η are described by a $U(2)$ matrix

$$\Sigma = \exp\left\{\frac{\sqrt{2}i}{F_\pi}(\tau \cdot \pi + \eta)\right\}, \tag{3}$$

with $F_\pi = 132$ MeV. Chirally invariant Lagrangians can be written with the help of the square root of the Σ field, $\xi = \Sigma^{1/2}$. We define two combinations

3

$$p_\mu = \frac{i}{2}(\xi\partial_\mu\xi^\dagger - \xi^\dagger\partial_\mu\xi), \qquad v_\mu = \frac{i}{2}(\xi\partial_\mu\xi^\dagger + \xi^\dagger\partial_\mu\xi) \ . \tag{4}$$

The interaction of heavy mesons with the light ones is given by

$$\mathcal{L}_d = M\,d\,\mathrm{Tr}\left(H\gamma^\mu\gamma^5 p_\mu\bar{H}\right) \ . \tag{5}$$

The coupling constant d is not very well known; from a single pole fit to the data on $D \to K$ and $D \to K^*$ semileptonic decays [9], one obtains $d \approx 0.53$. A straight forward use of the static $SU(6)$, or more precisely $SU(4)$ symmetry gives $d = 1$; reducing this by the ratio $(g_A/g_V)/(5/6)$ results in $d = 0.7$. The value $d = 0.53$, which we shall use, is a conservative estimate. If we include the strange quark content of the η and $\eta - \eta'$ mixing the heavy meson-η coupling has to be reduced to d'; using the known $\eta - \eta'$ mixing angle of $11°$ we find that $d'/d \sim 0.4$.

The light vector meson fields, ρ_μ and ω_μ, are combined as

$$\rho_\mu = \frac{1}{\sqrt{2}}(\rho_\mu \cdot \tau + \omega) \ , \tag{6}$$

and we also define a field strength tensor

$$F_{\mu\nu}(\rho) = \partial_\mu\rho_\nu - \partial_\nu\rho_\mu - i\tilde{g}[\rho_\mu, \rho_\nu] \ . \tag{7}$$

The width of the ρ sets $\tilde{g} = 3.93$. The coupling of these vector mesons is accomplished by modifying [9] $\mathcal{L}_{free}$, Eq. (2) to

$$\mathcal{L}_{kinetic} = -iM\,V_\mu Tr\left[H\{\partial_\mu - i\alpha\tilde{g}\rho_\mu - i(1-\alpha)v_\mu\}\bar{H}\right] \ , \tag{8}$$

where v_μ is defined in Eq. (4). This term is invariant under chiral transformations. Vector dominance suggests that $\alpha \approx 1$, which is the value we use in this work. In addition we add a magnetic moment term

$$\mathcal{L}_c = \frac{-icM}{m_v}Tr\left[H\gamma^\mu\gamma^\nu F_{\mu\nu}(\rho)\bar{H}\right] \ . \tag{9}$$

m_v is a fiducial light vector meson mass, which appears from dimensional considerations. The coupling constant c is constrained to [10]

$$2.27 < c/d < 3.75\,. \tag{10}$$

Phenomenology of the nucleon-nucleon potential [11] requires the exchange of a $0^+(0^{++})$ particle with a mass of ~ 550 MeV. This represent the effective potential due to the exchange of two pions in box diagrams with isobars in the direct channel and the exchange of the broad $\pi - \pi$ resonance $f_0(980)$. In our case, by considering both the scalar and vector mesons in a multichannel formalism, the box diagrams are automatically included and the exchange of the $f_0(980)$ contributes to that part of the potential whose range is shorter than that which we consider explicitly; hence we will not include any $0^+(0^{++})$ exchanges.

B. Interaction potential

Using the coupling constants determined in the previous subsection it is straightforward to find the interaction potential. Let $\mathbf{S}^i$ denote the spin of the light quark in the heavy meson, $i = 1, 2$, while $\mathbf{S}$ and $\mathbf{I}$ denote the total spin and isospin of the two light quarks. The position space potential is a combination of central, spin-spin and tensor terms,

$$V(\mathbf{x}) = V_0(\mathbf{x}) + (\mathbf{S}^1 \cdot \mathbf{S}^2) V_S(\mathbf{x}) + \left((\mathbf{S}^1 \cdot \hat{\mathbf{x}})(\mathbf{S}^2 \cdot \hat{\mathbf{x}}) - \frac{1}{3} \mathbf{S}^1 \cdot \mathbf{S}^2 \right) V_T(\mathbf{x})\,, \tag{11}$$

where

$$V_0(\mathbf{x}) = \frac{\tilde{g}^2}{4\pi} \left[(\mathbf{I}^2 - \frac{3}{2}) \frac{e^{-m_\rho r}}{r} + \frac{1}{2} \frac{e^{-m_\omega r}}{r} \right]\,,$$

$$V_S(\mathbf{x}) = \frac{d^2}{\pi F_\pi^2} \left[(\mathbf{I}^2 - \frac{3}{2}) Y(r, m_\pi) + \frac{1}{2} \frac{d'^2}{d^2} Y(r, m_\eta) \right] + \frac{4c^2}{\pi m_v^2} \left[(\mathbf{I}^2 - \frac{3}{2}) 2Y(r, m_\rho) + Y(r, m_\omega) \right]\,,$$

$$V_T(\mathbf{x}) = \frac{d^2}{\pi F_\pi^2} \left[(\mathbf{I}^2 - \frac{3}{2}) F(r, m_\pi) + \frac{1}{2} \frac{d'^2}{d^2} F(r, m_\eta) \right] - \frac{4c^2}{\pi m_v^2} \left[(\mathbf{I}^2 - \frac{3}{2}) F(r, m_\rho) + \frac{1}{2} F(r, m_\omega) \right]\,, \tag{12}$$

and

$$F(r, m) = e^{-mr} \left(\frac{3}{r^3} + \frac{3m}{r^2} + \frac{m^2}{r} \right)\,,$$

$$Y(r, m) = \frac{m^2}{3r} e^{-mr}\,. \tag{13}$$

5

The $1/r^3$ part of the tensor force, V_T, is too singular to permit a solution of the resulting Schrödinger equation; in addition there is a delta function at the origin lurking in that potential. As we have not taken into account the exchange of mesons whose mass is greater than that of the light vector mesons we will, as in Ref. [5], flatten the potentials at some r_c; we will choose $r_c = 1/m_\rho$; we will discuss how the results depend on r_c for $r_c > 1/m_\rho$.

III. BOUND STATE CONFIGURATIONS

In order to obtain a bound state we require that the long range part of the potential (π exchange) be attractive. From Eqs. (11,12) we see that the light quark configurations must be either $\mathbf{I} = 1$, $\mathbf{S} = 0$ or $\mathbf{I} = 0$, $\mathbf{S} = 1$; as expected the greatest attraction is in an orbital S-wave state. As the space-spin-isospin of the light quarks is antisymmetric they must be in a symmetric color state, the $\mathbf{6}$; this implies that the two $\bar{b}$ quarks are in the $\mathbf{6}^*$, forcing them to be in a spin singlet state. Thus the possible bound states are either a $I = 1$, $J = 0$ state or a $I = 0$, $J = 1$ state. These are linear combinations of B's and B^*'s:

$$
\begin{aligned}
|I = 1, I_3; J = 0\rangle &= \frac{1}{2}\langle 1, I_3|1, m; 1, m'\rangle \\
&\quad \left(|B^{*m}_{+1} B^{*m'}_{-1}\rangle + |B^{*m}_{-1} B^{*m'}_{+1}\rangle - |B^{*m}_0 B^{*m'}_0\rangle + |B^m B^{m'}\rangle \right), \\
|I = 0; J = 1, J_3 = 1\rangle &= \frac{1}{2}\langle 0, 0|1, m; 1, m'\rangle \\
&\quad \left(|B^{*m}_{+1} B^{*m'}_0\rangle + |B^{*m}_0 B^{*m'}_{+1}\rangle - |B^{*m}_{+1} B^{m'}\rangle + |B^m B^{*m'}_{+1}\rangle \right),
\end{aligned}
\tag{14}
$$

with similar expressions for the other two $J = 1$ states.

The expectation value of the mass operator in these states is twice the spin averaged mass

$$
\langle \mathcal{M} \rangle = 2M_{\text{eff}} = 2\,\frac{3M_{B^*} + M_B}{4} \, .
\tag{15}
$$

Eqs. (14) tell us into what combinations of B's and B^*'s these states may decay.

IV. SCHRÖDINGER EQUATIONS AND SOLUTIONS

A. Eigenvalue problem

To order $1/M$ the Hamiltonian of this system is

$$H = 2M_{\text{eff}} - \frac{1}{M_{\text{eff}}} \nabla^2 + V(\mathbf{x}) \,. \tag{16}$$

A basis for the system is provided by the states $|II_3\rangle \, |SS_3\rangle \, |lm\rangle$, where l, m are the orbital angular momentum quantum numbers of the bound system. Note that l, m includes the orbital angular momentum of the heavy quarks as well. Since the potential is rotationally invariant, the total angular momentum

$$\mathbf{J} = \mathbf{S} + \mathbf{L} \tag{17}$$

is a good quantum number and can be used to label the eigenstates. The $J = 0$ (singlet) state is written as

$$\Psi_{J=0} = \frac{\psi(r)}{r} \, Y_{00} \, |s = 0\rangle \,. \tag{18}$$

The $J = 1$ (triplet) state is a linear combination of $|S = 1, \, l = 0\rangle$ and $|S = 1, \, l = 2\rangle$ and can be written as

$$\Psi_{J=1}^{J_3} = \frac{u(r)}{r} \, \mathcal{Y}_{1;0,1}^{J_3} + \frac{w(r)}{r} \, \mathcal{Y}_{1;2,1}^{J_3} \,, \tag{19}$$

where $\mathcal{Y}_{J;l,S}^{J_3}$ is a generalized spherical harmonic obtained by combining spin and orbital angular momentum. Both the singlet and triplet states must further be supplemented by an isospin label, with $I = 0$ or 1.

With $\epsilon = E - 2M_{\text{eff}}$, the eigenvalue equation for the singlet case is

$$\psi'' = M_{\text{eff}} \left(V_0 - \frac{3}{4} V_S - \epsilon \right) \psi \,; \tag{20}$$

prime denotes differentiation with respect to r. For the spin triplet case the tensor force couples $l = 0$ to $l = 2$ and we get two coupled differential equations,

$$u'' = M_{\text{eff}} \left[\left(V_0 + \frac{1}{4} V_S - \epsilon \right) u + \frac{1}{3\sqrt{2}} V_T w \right] \,,$$
$$w'' = M_{\text{eff}} \left[\left(V_0 + \frac{1}{4} V_S + \frac{6}{M_{\text{eff}} r^2} - \frac{1}{6} V_T - \epsilon \right) w + \frac{1}{3\sqrt{2}} V_T u \right] \,. \tag{21}$$

7

B. Numerical results

We now consider the numerical solutions to Eqs. (20) and (21) for the energy eigenvalue ϵ; d is fixed at 0.53 and the B, B^* masses are taken to be 5.279 GeV and 5.325 GeV respectively. c/d is varied between 2.27 and 3.7.

For the singlet case, we find that there are bound states only in the $I = 1$ channel, forcing the color wavefunction to be in the symmetric **6** representation, as mentioned earlier. We observe from the decomposition Eq. (14) of the singlet wavefunction that the lightest constituents the state can decay into is two B mesons. Hence the true binding energy is

$$\epsilon_B = 2M_{\text{eff}} - 2M - |\epsilon|$$
$$= 69 \text{ MeV} - |\epsilon| ; \tag{22}$$

the state is bound if ϵ_B is negative. Binding occurs only for $c/d \geq 3.6$ and $\epsilon_B = 40$ GeV at the extreme of the allowed range, $c/d = 3.7$. Hence we conclude that the singlet configuration is unlikely to be bound.

We now move to the triplet configuration. Here, the lightest constituents the state can decay into is a B and a B^* meson. hence the true binding energy is given by

$$\epsilon_B = 2M_{\text{eff}} - M - M^* - |\epsilon|$$
$$= 23 \text{ MeV} - |\epsilon| . \tag{23}$$

Fig. 1 summarizes the numerical results for the allowed range of c/d. It is clear from the figure that there is a stable bound state over the entire allowed range of the coupling constant c.

One interesting issue to explore is the sensitivity of the binding energy to the cutoff r_c, below which the potential is flattened. Fig. 2 shows the dependence of ϵ_B on r_c, for a central value of the ratio $c/d = 3$. The lightest particle above the vector mesons which can contribute to the potential is the $f_0(980)$, or the $\eta'(958)$. These correspond to a value of r_c in the neighborhood of about 1 GeV^{-1}. We see from Fig. 2 that a cutoff less than

about 1 GeV^{-1} leads to a very high binding energy; indeed, the non-relativistic formulation of the system breaks down. Hence the potential derived here cannot be trusted for values of the cutoff much less than 1 GeV^{-1}. We note that $r_c = 1/m_\rho$ is a conservative estimate for determining whether binding does or does not occur.

V. $D - D$ AND $D - B$ SYSTEMS

We repeated the calculations for smaller reduced masses to see what happens for the $D - D$ and $D - B$ systems. From Fig. 3 we could claim that both of these will be bound for a triplet light quark spin configuration. This interpretation is too naive. As has been emphasized in Ref. [4] the heavy quark limit cannot be use for the study of binding of charmed mesons. For π exchange the range of the potential is nor $1/m_\pi$, but $1/\mu$ with $\mu = \sqrt{m_\pi^2 - (M_D^* - M_D)^2}$. This value is close to zero, and for several charge combinations negative. (For the $B - B$ system this gives a 6% correction to the range of the π exchange potential.) Thus the breaking of heavy quark symmetry and even the breaking of isospin invariance will be crucial to the study of this system. As in the nucleon-nucleon case we could look at the single channel $D - D$ problem. As there is no one-π exchange potential for this configuration, this system is unlikely to be bound. The $D - D^*$ mass difference would force us to treat this as a three body $D - D - \pi$ system.

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FIGURES

FIG. 1. Binding energy, ϵ_B, of the $I = 0$, $J = 1$ state as a function of the magnetic vector meson coupling c/d.

FIG. 2. Binding energy, ϵ_B, of the $I = 0$, $J = 1$ state as a function of the cutoff for $c/d = 3.03$.

FIG. 3. Eigenvalue of the Schrödinger Equation, ϵ, of the $I = 0$, $J = 1$ state as a function of the reduced mass for $c/d = 3.03$ and $r_c = 1/m_\rho$.

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Fig. 3

Fig. 2

Fig. 1